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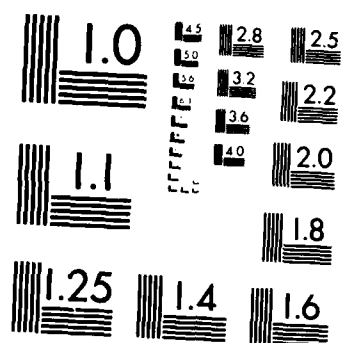
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Dr. Emmanuel N. Barron

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In this project the rigorous connection between viscosity solutions for the Bellman equation in optimal control and the Pontryagin Maximum Principle has been established. The method developed for controlled ordinary differential equations was extended to infinite dimensions to derive the Pontryagin principle for 1) a class of controlled nonlinear evolution equations in a Hilbert space, 2) a class of controlled nonlinear, divergence form parabolic partial differential equations; and 3) a class of differential-difference equations.

Several additional subjects were studied. Namely, the extension of the idea of viscosity solution to equations with only time-measurable Hamiltonians; the optimal cooling of a free boundary problem with Stefan problem dynamics.

Finally, two problems of interest in specific applications were solved. First, we explicitly characterized the optimal control in the class of monotone functions which minimizes the

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$H_v^1$  distance to a given function. This problem, a specific "monotone follower" problem, arises in production planning. ~~Second, we considered~~ an optimal portfolio selection problem which includes stock, options, bonds and borrowed cash at an interest rate different from the bond interest rate. ~~We formulated~~ this problem using stochastic optimal control and explicitly constructed the solution of the Bellman equation. The objective of the study was to derive the option price which the market sets to minimize the investor's maximal expected utility of wealth.

VISCOSITY METHODS IN OPTIMAL CONTROL OF DISTRIBUTED SYSTEMS

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This project has resulted in the following publications or manuscripts submitted for publication:

1. "The Pontryagin maximum principle from dynamic programming and viscosity solutions to first order partial differential equations," Trans. Amer. Math. Soc., 298 (1986), 635-641. (with R. Jensen)
2. "The necessary conditions for optimal control in Hilbert spaces," J. Math. Anal. Appls., to appear. (with V. Barbu and R. Jensen)
3. "Bang-bang controllers for an optimal cooling problem," Control and Cybernetics, to appear. (with V. Barbu)
4. "Minimizing a quadratic payoff with monotone controls," Math. of Operations Research,
5. "Generalized viscosity solutions for Hamilton-Jacobi equations with time-measurable Hamiltonians," J. Differential Equations. 68 (1987), 10-21. (with R. Jensen)
6. "Application of viscosity solutions of infinite dimensional Hamilton-Jacobi-Bellman equations to some problems in distributed optimal control," submitted.
7. "A stochastic control approach to the pricing of options," submitted. (with R. Jensen)

DESCRIPTION OF RESULTS

The major result obtained in this project was the *rigorous* derivation of the Pontryagin maximum principle using dynamic programming. A *formal* derivation has been known for over thirty years but to make it rigorous had required that the value function

be twice continuously differentiable, which it generally is not. We were able to overcome this difficulty by exploiting the idea of viscosity solutions of the Bellman equation introduced by M.G.Crandall and P-L.Lions in 1983. The method of proof we used in [1] was also exploited in [2] and [6] to derive the Pontryagin principle for some distributed optimal control problems. In [2] we considered a class of nonlinear optimal control problems governed by evolution equations in a Hilbert space. In [6] we derived the Pontryagin principle for (i) ordinary differential equations with a delay and (ii) nonlinear parabolic partial differential equations in divergence form.

A problem which arose when we were working on [1] was the fact that the standard theory of viscosity solutions required the Hamiltonian to be a continuous function of time. In [1] this was not satisfied. Therefore, we extended the ideas of viscosity solutions to the time-measurable case in order to remove the continuity restriction. We published the result in [5]. It turns out that H.Ishii had also considered this problem and solved it by a method completely different from ours.

The paper [3], with V.Barbu, considered the optimal control of a free boundary problem. The dynamics consisted of the Stefan problem governing the physics of a body of ice in contact with a region of water with controlled cooling. This problem was solved by converting the dynamics to a controlled variational inequality and applying the necessary conditions derived by Barbu for such problems. We then formulated several optimization problems of interest and characterized the optimal control as bang-bang.

Many problems of interest involve the use of *monotone controls*. In the literature this is known as the "monotone follower problem" or problems with "no turning back." In [4] we consider the problem of minimizing the distance of a monotone function to a given function where the distance is measured by the squared integral distance to the function and the derivative of the function, (i.e. the  $H^1$  norm). This problem arises in many problems of interest in production planning. We solve this problem by *explicitly* deriving the optimal control. That is, we construct the optimal control by solving a nonlinear second order ordinary differential equation.

Finally, we worked on an application of stochastic optimal control which arises in financial economics. That is, in [7] we constructed a stochastic optimal control model of a portfolio consisting of stock, bonds, borrowed money, and call options on the stock. The goal of the investor is to maximize his expected utility

of wealth at expiration of the options. The goal of the paper was to determine the option price which minimizes the investor's maximal expected utility of wealth. This goal was accomplished and many results of interest were derived in comparing our option price with the classically derived Black-Scholes price.

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